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We show that for the S-state π^0 -production in processes $\gamma + d \rightarrow d + \pi^0$ and $e^- + d \rightarrow e^- + d + \pi^0$ the rescattering effects due to the transition: $\gamma + d \rightarrow p + p + \pi^-$ (or $n + n + \pi^+$) $\rightarrow d + \pi^0$ are cancelled out due to the Pauli principle. The large values for these effects predicted in the past may result from the fact that the spin structure of the corresponding matrix element and the necessary antisymmetrization induced by the presence of identical protons (or neutrons) in the intermediate state was not taken into account accurately. One of the important consequences of these considerations is that π^0 photo- and electro-production on deuteron near threshold can bring direct information about elementary neutron amplitudes.

I. INTRODUCTION

A considerable experimental and theoretical activity has been going on in the field of near threshold pion production in γN - and eN - collisions. Recently, new results have been obtained for π^0 -production on protons, $\gamma + p \rightarrow p + \pi^0$, using tagged photons [1,2]:

- the discovery of a unitary cusp in the energy dependence of the E_{0+} amplitude for $\gamma + p \rightarrow p + \pi^0$, near the reaction threshold,
- the contradiction of the measured value of $E_{0+}(p\pi^0)$ with the predictions of "old" low energy theorems [3].

The Chiral Perturbation Theory (ChPT) [4] was very successful in the explanation of the properties of different S- and P-wave multipole amplitudes for $\gamma + p \rightarrow p + \pi^0$ in the near threshold region. The process of π^0 electro-production on protons [5–7], $e^- + p \rightarrow e^- + p + \pi^0$, opens also new interesting possibilities due to the longitudinal polarization of the virtual photon and the non-trivial dependence of the multipole amplitudes on the momentum transfer squared, Q^2 from the initial to the scattered electron. But last data [7] show a serious discrepancy with the calculation in the framework of heavy baryon ChPT [8].

For a further test of different models, which can be applied to pion photo- and electro-production, the information about the amplitudes of the processes $\gamma + n \rightarrow n + \pi^0$ and $e^- + n \rightarrow e^- + n + \pi^0$ is essential. The deuteron processes $\gamma + d \rightarrow d + \pi^0$ and $e^- + d \rightarrow e^- + d + \pi^0$ seem well adapted for this aim, as, in impulse approximation (IA), the corresponding matrix elements are determined by the coherent sum of amplitudes for elementary processes $\gamma(\gamma^*) + p \rightarrow p + \pi^0$ and $\gamma(\gamma^*) + n \rightarrow n + \pi^0$ [9–11], where γ^* is the virtual photon. However, since, in the threshold region, these amplitudes are small in comparison with the amplitudes of π^\pm photoproduction on nucleons, rescattering effects, (RE) due to the following two-step processes:

$$\gamma + d \rightarrow p + p + \pi^- \quad (n + n + \pi^+) \rightarrow d + \pi^0 \quad (1)$$

can be, in principle, strongly competitive with the direct π^0 -production from a single nucleon [12–19]. If it is the case, the reaction $\gamma + d \rightarrow d + \pi^0$ can not allow a direct and model independent extraction of the elementary neutron amplitudes.

A dedicated experiment [20], with a tagged photon beam, was devoted to measurements of the cross section and of the angular distribution for the inclusive π^0 -production in the $d(\gamma, \pi^0)X$ reaction in the near threshold region. Inelastic contributions, due to the deuteron disintegration, $\gamma + d \rightarrow \pi^0 + n + p$ could not be resolved in the experiment, and their contribution was estimated through a theoretical model. It was shown that the electric dipole amplitude E_{0+}

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for the $\gamma + d \rightarrow \pi^0 + d$ process (which is proportional to the amplitude referred later on as g_e) can be determined with good accuracy. The data are characterized by a large backward-forward asymmetry of the angular π^0 -distribution. The deduced E_{0+} amplitude, has a negative sign, relative to the definite combination of P-wave amplitudes, which is determined from a theory for $\gamma + N \rightarrow N + \pi$. The E_{0+} amplitude for $\gamma + d \rightarrow d + \pi^0$ is certainly sensitive to the E_{0+} amplitude for $\gamma + n \rightarrow \pi^0 + n$, but the quantitative determination of this last amplitude depends essentially on RE. The negative sign for $E_{0+}(\gamma d \rightarrow d\pi^0)$ was considered as a confirmation of the validity of ChPT predictions for $\gamma + N \rightarrow N + \pi^0$, and of the important role of RE.

Coherent π^0 threshold electro-production on the deuteron at $Q^2=0.1 \text{ GeV}^2$ has been studied by the A1 collaboration at the Mainz Microtron MAMI, with $d\pi^0$ -excitation energy up to $\Delta W = 4 \text{ MeV}$ [21]. The longitudinal threshold amplitude, which was extracted by the Rosenbluth fit is smaller (in absolute value) by a factor two than the value predicted by ChPT calculations. The discrepancy at the level of the cross section is of one order of magnitude.

Note, in this connection, that the existing calculations of RE give very different quantitative predictions. These effects are very sensitive to many ingredients of the corresponding model, as, for example, the short distance behavior of the deuteron wave function, the shape of the pion propagator in the intermediate state, the choice of the operator for the elementary process $\gamma + N \rightarrow N + \pi^0$, the procedure of integration (for example in [9] a six fold integration was done) etc. The accuracy of assumptions and simplifications in the theoretical calculations can not be easily controlled. This may be the reason for which large discrepancies exist in the theoretical results.

Our main aim here is to demonstrate that for the threshold S-state π^0 -meson production in processes $\gamma + d \rightarrow d + \pi^0$ and $e^- + d \rightarrow e^- + d + \pi^0$ the application of Pauli principle and the conservation of P-parity and total angular momentum induces an exact cancellation of RE due to the reactions $\gamma + d \rightarrow p + p + \pi^-$ (or $n + n + \pi^+$) $\rightarrow d + \pi^0$.

This paper is organized as follows. In Section II we establish the spin structure of the threshold amplitudes for $\gamma + d \rightarrow d + \pi^0$ and analyze the most simple polarization phenomena for this process. The cancellation of RE, due to the processes $\gamma + d \rightarrow p + p + \pi^-$ ($n + n + \pi^+$) $\rightarrow d + \pi^0$ is considered in Section III. The description of the process $\gamma + d \rightarrow p + p + \pi^-$ in the framework of impulse approximation is presented in Section IV. In Section V we consider the general kinematics of the threshold inelastic π -meson photoproduction, $\gamma + d \rightarrow N + N + \pi$ for charged and neutral pions. Finally, in Section VI we consider the 'scalar' deuteron photoproduction, $\gamma + d \rightarrow (n + p)_s + \pi^0$, (i.e. the np -system is in a singlet state) where RE must be important.

II. SPIN STRUCTURE OF THRESHOLD AMPLITUDES FOR $\gamma + D \rightarrow D + \pi^0$

Let us consider, firstly, the simplest process of pion photoproduction on the deuteron, $\gamma + d \rightarrow d + \pi^0$, in the threshold region, where the main contributions are due to π^0 production in S- and P-states. For S-state production (i.e. with $\mathcal{J}^P = 1^-$, where \mathcal{J} is the total angular momentum of the produced $d\pi^0$ -system and P is its P -parity) the conservation of P -parity and total angular momentum allows two multipole transitions, $E1$ and $M2 \rightarrow \mathcal{J}^P = 1^-$, with the following parametrization of the spin structure of corresponding matrix element:

$$\mathcal{M}(\gamma d \rightarrow d\pi^0) = g_e \vec{e} \cdot \vec{D}_1 \times \vec{D}_2^* + g_m (\vec{e} \times \vec{k} \cdot \vec{D}_1 \vec{k} \cdot \vec{D}_2^* + \vec{e} \times \vec{k} \cdot \vec{D}_2^* \vec{k} \cdot \vec{D}_1), \quad (2)$$

where \vec{e} is the photon polarization vector, \vec{D}_1 and \vec{D}_2 are the axial vectors of deuteron polarization (in initial and final states), \vec{k} is the unit vector along the three-momentum of photon in the center of mass (CMS) of the considered reaction, g_e and g_m are the multipole amplitudes, describing the $E1$ - and $M2$ -photon absorption with the S-state π^0 -production. In the general case these amplitudes are complex functions of the photon energy, E_γ , but due to the T-invariance of hadron electrodynamics [22], the relative phase of multipole amplitudes must be equal to 0 or π .

The differential cross section for the process $\gamma + d \rightarrow d + \pi^0$ is expressed in terms of the multipole amplitudes g_e and g_m , in a particular normalization, as:

$$\left(\frac{d\sigma}{d\Omega} \right)_0 = \frac{2}{3} (|g_e|^2 + |g_m|^2).$$

For S-wave pion production, among all possible one-spin polarization observables, only the tensor analyzing power \mathcal{A} (in $\gamma + \vec{d} \rightarrow d + \pi^0$) does not vanish:

$$\frac{d\sigma}{d\Omega}(\gamma \vec{d}) = \left(\frac{d\sigma}{d\Omega} \right)_0 [1 + (Q_{ab} k_a k_b) \mathcal{A}], \quad (3)$$

where Q_{ab} is the tensor polarization of deuteron target. The corresponding density matrix can be written as:

$$\overline{D_{1a} D_{1b}^*} = \frac{1}{3} (\delta_{ab} - \frac{3}{2} i \epsilon_{abc} S_c - Q_{ab}), \quad Q_{ab} = Q_{ba}, \quad Q_{aa} = 0,$$

where \vec{S} is the vector deuteron polarization. Averaging over the photon polarizations and summing over the final deuteron polarizations, one finds:

$$\mathcal{A} = -\frac{1}{4} \frac{|g_e|^2 + |g_m|^2 + 6\mathcal{R}e\ g_e g_m^*}{|g_e|^2 + |g_m|^2} = -\left(\frac{1}{4} \pm \frac{3}{2} \frac{r}{1+r^2}\right), \quad (4)$$

where we defined the ratio $r = |g_m|/|g_e|$. The sign \pm corresponds to the two possible relative signs of the amplitudes g_e and g_m . The behavior of the asymmetry \mathcal{A} as a function of r is shown in Fig. 1. One can see the large sensitivity of \mathcal{A} to the relative sign and to the ratio of the g_e and g_m amplitudes, even at small r .

The complete experiment, i.e. the full determination of the amplitudes g_e and g_m , is realized, at threshold, through the measurement of two observables only, the differential cross section and the tensor analyzing power:

$$|g_e + g_m|^2 = (1 - 2\mathcal{A}) \left(\frac{d\sigma}{d\Omega}\right)_0, \quad |g_e - g_m|^2 = 2(1 + \mathcal{A}) \left(\frac{d\sigma}{d\Omega}\right)_0.$$

The interference contribution, which is sensitive to the small magnetic amplitude g_m , can be determined through the following formula:

$$g_e g_m = -\frac{1}{4}(1 + 4\mathcal{A}) \left(\frac{d\sigma}{d\Omega}\right)_0.$$

Note that all the polarization phenomena for the process $\gamma + d \rightarrow d + \pi^0$ near threshold can be predicted in terms of the tensor analyzing power and of the differential cross section. To demonstrate this, let us consider, as an example, the collision of polarized photons with a polarized deuteron target. In case of a linearly polarized photon beam, we can define the following asymmetry:

$$\Sigma = \frac{d\sigma(e_x) - d\sigma(e_y)}{d\sigma(e_x) + d\sigma(e_y)}$$

where e_x and e_y are the components of the photon polarization vector in a coordinate system with the z -axis along the 3-vector \vec{k} .

The asymmetry Σ can be written as a function of the multipole amplitudes g_e and g_m , and of the tensor polarizations as:

$$\Sigma = \frac{(Q_{xx} - Q_{yy})|g_e - g_m|^2}{(|g_e|^2 + |g_m|^2)(4 - Q_{zz}) - 6g_e g_m Q_{zz}},$$

so that:

$$\Sigma \frac{d\sigma}{d\Omega}(\gamma \vec{d}) = 2(Q_{xx} - Q_{yy})(1 + \mathcal{A}) \left(\frac{d\sigma}{d\Omega}\right)_0.$$

In IA, for $\gamma + d \rightarrow d + \pi^0$, (Fig. 2), the amplitude g_e , which is generated by the S-wave component of the deuteron wave function (at relatively small internal momentum), is proportional to the sum of E_{0+} amplitudes for $\gamma + p \rightarrow p + \pi^0$ and $\gamma + n \rightarrow n + \pi^0$ -processes:

$$g_e = F_s(t) \left[E_{0+}^{p\pi^0} + E_{0+}^{n\pi^0} \right], \quad (5)$$

where $F_s(t)$ is the S-wave deuteron form factor at $t = -E_{th}^2$ and E_{th} is the photon threshold energy (in CMS). The amplitude g_m of magnetic quadrupole absorption has not an analogue for the elementary processes $\gamma + N \rightarrow N + \pi$, at threshold. In the framework of IA, it can not be derived from the spin structure $\vec{\sigma} \cdot \vec{e}$ for the threshold amplitude of the elementary process $\gamma + N \rightarrow N + \pi$. Therefore the amplitude g_m is very sensitive to the details of the reaction mechanism, in $\gamma + d \rightarrow d + \pi^0$. The realization of the complete experiment, as indicated above, would be very interesting, in this respect.

III. CANCELLATION OF RESCATTERING CONTRIBUTIONS

The impulse approximation is only one of the ingredients in the analysis of the process $\gamma(\gamma^*) + d \rightarrow d + \pi^0$. Let's discuss now RE.

It is well known ([12,13] and refs. herein) that the E_{0+} amplitude for the charged pion production on nucleons is larger (in absolute value) than the corresponding amplitude for neutral pion production: $|E_{0+}^{n\pi^+}| \simeq 20|E_{0+}^{p\pi^0}|$. This fact is generally accepted as the underlying reason for the manifestation of RE due to reactions (1). In this case, a model independent information about the elementary amplitude of π^0 -production on neutron, $\gamma + n \rightarrow n + \pi^0$ can not be derived from the study of the reaction $\gamma + d \rightarrow d + \pi^0$. Previous calculations [12–14] have shown that RE, which involve intermediate charged pions, essentially change the predictions of the impulse approximation.

We show here that RE due to reactions (1), cancel out for the threshold amplitudes, if one takes into account the spin structure of the corresponding transitions which are allowed by the Pauli principle and the conservation of angular momentum and P-parity. As a result, we will prove that the state with $\mathcal{J}^P = 1^-$ is forbidden for the intermediate $pp\pi^-$ (and $nn\pi^+$)-system, if $\ell_1 = \ell_2 = 0$ (threshold conditions), where ℓ_1 is the orbital angular momentum for the pp -system, and ℓ_2 is the pion orbital momentum relative to the pp -system (Fig. 3). Therefore reactions (1) can not occur in threshold regime.

For the pp -system with $\ell_1 = 0$, only the singlet state is allowed. Therefore, at threshold of $\gamma + d \rightarrow p + p + \pi^-$, \mathcal{J}^P takes the value 0^- (instead of $\mathcal{J}^P = 1^-$ for the threshold $d + \pi^0$ -system). This is illustrated also in the corresponding Feynman diagrams (Fig. 4), where at threshold, we have for the three-momentum of the protons: $\vec{p}_1 = \vec{p}_2 = 0$, with evident cancellation of the two contributions.

This result is valid for any parametrization of the deuteron wave function and relative value of the amplitudes for the different pion production processes. It is also correct for coherent π^0 electro-production on deuteron, $e + d \rightarrow e + d + \pi^0$, at any value of momentum transfer square Q^2 , in the space-like region and for any polarization (transversal and longitudinal) of virtual photons. The main assumption, done here, is that both nucleons in the intermediate state are on mass shell, so that the pp -system, with $\ell_1 = 0$ has positive P-parity. Off-shell protons would have an antinucleon component with negative P-parity. In principle, these intermediate configurations can contribute, but they have been neglected in all previously quoted calculations of RE, which were done in framework of non-relativistic approach.

The exact cancellation of RE in the S^- state, for $\gamma + d \rightarrow d + \pi^0$ at threshold is a rigorous general result, which has to be verified by any model calculation. But, technically, this can be a difficult problem. To show this, let us consider the standard procedure of RE calculation, with a single πN -scattering. To satisfy the Pauli principle, (for the intermediate πNN -state, with two identical nucleons), it is necessary to add to the usual diagram (where a pion, photoproduced on one nucleon, is scattered by another one, Fig. 5a), the diagram (5b), where the pion is scattered by the same nucleon.

Only the sum of (a)+(b) contributions, calculated with the same vertexes, satisfies the Pauli principle, resulting in an exact compensation of RE. But the (5b) contribution, is a particular part of the amplitude for $\gamma + d \rightarrow d + \pi^0$, calculated in IA (Fig. 6). So, to avoid a double counting, in model calculations it is necessary to subtract from the full IA $\gamma + N \rightarrow N + \pi^0$ amplitude, the important part due to pion rescattering on the same nucleon: $\gamma + p \rightarrow n + \pi^+ \rightarrow p + \pi^-$. This means that the IA amplitude for $\gamma + d \rightarrow d + \pi^0$ must be calculated with a *renormalized* amplitude for $\gamma + p \rightarrow p + \pi^0$, (Fig. 7), not with the standard one. This renormalization procedure is nontrivial and looks like a numerical artifact, but the calculations of RE, due to (5a) only, violate the Pauli principle and result in large RE for $\gamma + d \rightarrow d + \pi^0$.

Therefore, the most delicate problem in calculating the contributions (5a) and (5b), is to satisfy the Pauli principle and to avoid double counting, which can induce large, non-physical RE. Let us reanalyze, at the light of the previous discussion, the available experimental data. The value of the threshold amplitude, extracted from deuteron photoproduction data [20], is $E_d = (-1.45 \pm 0.09) \times 10^{-3}/m_\pi$. More exactly, the experimental observable is the cross section, which is proportional to the amplitude squared. The minus sign has been attributed in order to be consistent with the ChPT predictions [16]. In the framework of IA, 5, if we assume that RE are absent, it is straightforward to extract the neutron amplitude. In Table I we report the values of the neutron amplitude for the two possible signs of the deuteron amplitude. We remark that the result obtained for $E_{0+}^{n\pi^0}$, using a positive value for E_d and the experimental value of $E_{0+}^{p\pi^0} = -1.13$ [1], is not far from the ChPT prediction.

The exact cancellation of RE's at the threshold of the process $\gamma + d \rightarrow d + \pi^0$, can explain naturally and in a model independent way, the absence of unitary cusp in the energy dependence of the E_{0+} -amplitude for this process recently experimentally observed [20]. This cusp is present on π^0 -photoproduction on the nucleon, due to the $\gamma + p \rightarrow n + \pi^+ \rightarrow p + \pi^0$ rescattering and they have been observed on a proton target [1,2], but the Pauli principle forbids the corresponding intermediate states in (1) (in case of a deuteron target). Therefore the absence of cusp can be considered as an experimental evidence of the cancellation of RE in threshold π^0 photoproduction on deuteron.

Note also that the experimental data about coherent π^0 electro-production on the deuteron at $Q^2=0.1$ GeV², do not show either any evidence of the corresponding cusp at 2.2 MeV above the π^0 -threshold [21]. This cusp should be present if RE, due to (1) were important.

Summarizing the previous discussion, the most crucial points in the evaluation of RE, especially in numerical calculations, in the near threshold region for $\gamma + d \rightarrow d + \pi^0$ are the following:

- cancellation of S-wave contributions (independently for $\pi^- pp$ and $\pi^+ nn$ intermediate states), which must be done analytically, in exact form;
- estimation of the relative role of other possible *non S-wave* contributions to the RE.

IV. ATTEMPT OF MULTIPOLE ANALYSIS

An threshold, i.e. when $\ell_1 = \ell_2 = 0$, for the process $\gamma + d \rightarrow p + p + \pi^-$ ($n + n + \pi^+$) we showed that RE cancel out. Other values of ℓ_1 and ℓ_2 are, in principle, possible, but their contribution can not be large, in the threshold region, due to centrifugal considerations. Let us consider $\ell_1 = \ell_2 = 1$ (Fig. 2), which is the next allowed possibility to obtain $\mathcal{J}^P = 1^-$, the threshold value. In order to generate such states, it is necessary to have a particular mechanism of RE. In the framework of the existing analysis of RE, based on the standard structure of the $\gamma + N \rightarrow N + \pi$ near-threshold amplitude, $E_{0+} \vec{\sigma} \cdot \vec{e}$, it is possible to show that two P-waves for the πNN - intermediate state are not allowed.

The matrix element for $\gamma + d \rightarrow p + p + \pi^-$ in the case of $\mathcal{J}^P = 1^-$, with $\ell_1 = \ell_2 = 1$ is proportional to the product of two small three-momenta: \vec{p} (proton) and \vec{q} (pion). Let us parametrize these contributions in a model independent way. The conservation of the total angular momentum and the P-parity allows the following multipole transitions for $\gamma + d \rightarrow \pi^- + p + p$ ($\mathcal{J}^P = 1^-$, $\ell_1 = \ell_2 = 1$): $E1$ and $M2 \rightarrow j = 0, 1$ and 2 , where j is the total angular momentum of the produced pp -system. Having $\ell_1 = 1$, such system has to be in triplet state, so $j = \vec{1} + \vec{1} = 0, 1, 2$.

The spin structure of these transitions is:

$$\chi_2^\dagger \vec{\sigma} \cdot \vec{p} \sigma_y \tilde{\chi}_1^\dagger \vec{q} \cdot \vec{e} \times \vec{D}, \quad E1 \rightarrow j = 0,$$

$$\chi_2^\dagger \left(\vec{\sigma} \cdot \vec{q} \vec{e} \times \vec{D} \cdot \vec{p} - \vec{\sigma} \cdot \vec{e} \times \vec{D} \cdot \vec{p} \cdot \vec{q} \right) \sigma_y \tilde{\chi}_1^\dagger, \quad E1 \rightarrow j = 1,$$

$$\chi_2^\dagger \left(\vec{\sigma} \cdot \vec{e} \times \vec{D} \cdot \vec{p} \cdot \vec{q} + \vec{\sigma} \cdot \vec{q} \vec{p} \cdot \vec{e} \times \vec{D} - \frac{2}{3} \vec{\sigma} \cdot \vec{p} \vec{q} \cdot \vec{e} \times \vec{D} \right) \sigma_y \tilde{\chi}_1^\dagger, \quad E1 \rightarrow j = 2,$$

$$\chi_2^\dagger \vec{\sigma} \cdot \vec{p} \sigma_y \tilde{\chi}_1^\dagger \left(\vec{e} \times \vec{k} \right) \times \vec{D} \cdot \vec{q}, \quad M2 \rightarrow j = 0,$$

$$\chi_2^\dagger \left(\vec{\sigma} \cdot \vec{q} p_a - \vec{\sigma}_a \vec{p} \cdot \vec{q} \right) \sigma_y \tilde{\chi}_1^\dagger \left[(\vec{e} \times \vec{k})_a \vec{k} \cdot \vec{D} + k_a \vec{e} \times \vec{k} \cdot \vec{D} \right], \quad M2 \rightarrow j = 1,$$

$$\chi_2^\dagger \left(\vec{\sigma}_a \vec{p} \cdot \vec{q} + p_a \vec{\sigma} \cdot \vec{q} - \frac{2}{3} q_a \vec{\sigma} \cdot \vec{p} \right) \sigma_y \tilde{\chi}_1^\dagger \left([\vec{e} \times \vec{k}]_a \vec{k} \cdot \vec{D} + k_a \vec{e} \times \vec{k} \cdot \vec{D} \right), \quad M2 \rightarrow j = 2.$$

Such spin structure can not be generated by the two diagrams (Fig. 4), which are typically used in the standard calculations of RE, as the sum of these diagrams is proportional to:

$$\begin{aligned} & \chi_2^\dagger \left[\vec{e} \cdot \vec{D}(u_s + u'_s) + \vec{e} \cdot \vec{p} \vec{p} \cdot \vec{D}(u_d + u'_d) \right. \\ & \left. + i \vec{\sigma} \cdot \vec{e} \times \vec{D}(u_s - u'_s) + \vec{p} \cdot \vec{D} \vec{\sigma} \cdot \vec{e} \times \vec{p}(u_d - u'_d) \right] \sigma_y \tilde{\chi}_1^\dagger, \end{aligned} \quad (6)$$

where u_s and u_d are two possible S- and D-components of the nonrelativistic deuteron wave function which depend on $|\vec{k} + \vec{p}|^2$, while u'_s and u'_d depend on $|\vec{k} - \vec{p}|^2$.

Therefore, in threshold region, where $\vec{p} \simeq 0$, $u_s - u'_s \rightarrow 0$, and $u_d - u'_d \rightarrow 0$, this mechanism can induce the following transitions for the $\pi^- pp$ system: $\mathcal{J}^P = 0^-$ with $\ell_1 = \ell_2 = 0$ and $\mathcal{J}^P = 2^-$ with $\ell_1 = \ell_2 = 0$ (both protons are in a singlet state). But $\mathcal{J}^P = 0^-$ is not a possible configuration for the reaction $\gamma + d \rightarrow d + \pi^0$ at threshold (as it was proved above), and $\mathcal{J}^P = 2^-$ corresponds to the D-wave of π^0 , with evidently small amplitudes. The NN -final interaction (Fig. 8a) can not transform a singlet pp -system to a triplet one, and πN rescattering (Fig. 8b) can not re-arrange the threshold spin structure of the matrix element for $\gamma + d \rightarrow p + p + \pi^-$ in the framework of the considered mechanism.

The possible triplet contributions to the matrix element (6), may appear only far from threshold, where $u_s \neq u'_s$ and $u_d \neq u'_d$: in this case the states with $\ell_1 = 1$ and 3 are possible, but then the P-parity of this channel is positive, because $\ell_2 = 0$, and, again, it is incompatible with $\mathcal{J}^P = 1^-$ (which characterizes the threshold conditions for $\gamma + d \rightarrow d + \pi^0$).

For a more general analysis, it is possible to take into account the full spin structure of the elementary process $\gamma + N \rightarrow N + \pi$ in the following form:

$$\vec{\sigma} \cdot \vec{e} f_1 + i\vec{e} \cdot \vec{k} \times \vec{q} f_2 + \vec{e} \cdot \vec{q} \vec{\sigma} \cdot \vec{k} f_3 + \vec{e} \cdot \vec{q} \vec{\sigma} \cdot \vec{q} f_4.$$

One can see that in threshold conditions for the pp -system, at this vertex all configurations for any value of ℓ_2 are allowed, but there is a restriction on ℓ_1 : only singlet pp -states with $\ell_1 = 0$ or $\ell_1 = 2$ are permitted. And only one configuration with a combination of $\ell_2 = 2$ and $\ell_1 = 2$ can produce $\mathcal{J}^P = 1^-$. However this is a small contribution, which results from the multiplication of at least 3 small factors: *D - wave of deuteron* \otimes *D - wave in* $\gamma + N \rightarrow N + \pi$ \otimes *D - wave in threshold* $N + N - \text{system}$.

Note that P-wave in $\gamma + N \rightarrow N + \pi^+$ combined with the effect due to $u_s - u'_s$ (or $u_d - u'_d$) can result in $\mathcal{J}^P = 1^-$. But we showed that this effect is small and, moreover, it is not related to the fact that $E_{0+}^{n\pi^+} \simeq 20 E_{0+}^{p\pi^0}$, which is commonly taken as an evidence of large RE. The standard mechanism of pion rescattering in the reactions $\gamma + d \rightarrow p + p + \pi^- \rightarrow d + \pi^0$, cannot produce large multipole amplitudes with $\mathcal{J}^P = 1^-$.

A similar analysis of RE holds also for threshold π^0 electro-production, $e + d \rightarrow e + d + \pi^0$. Let us mention in this respect, that the threshold matrix element for $\gamma^* + d \rightarrow d + \pi^0$ contains the contributions of the 3 following multipole transitions: $E1_t$, $E1_\ell$ and $M2 \rightarrow \mathcal{J}^P = 1^-$, where the indexes t and ℓ correspond to the absorption of a virtual photon with transversal and longitudinal polarization. Therefore the matrix element for the process $\gamma^* + d \rightarrow d + \pi^0$, corresponding to S-state π^0 -production has the following expression [23]:

$$\begin{aligned} \mathcal{M}(\gamma^* + d \rightarrow d + \pi^0) = & g_t(k^2)(\vec{e} \cdot \vec{D}_1 \times \vec{D}_2^* - \vec{e} \cdot \vec{k} \vec{k} \cdot \vec{D}_1 \times \vec{D}_2^*) + \\ & + g_\ell(k^2)\vec{e} \cdot \vec{k} \vec{k} \cdot \vec{D}_1 \times \vec{D}_2^* + \\ & + g_m(k^2)(\vec{e} \times \vec{k} \cdot \vec{D}_1 \vec{k} \cdot \vec{D}_2^* + \vec{e} \times \vec{k} \cdot \vec{D}_2^* \vec{k} \cdot \vec{D}_1), \end{aligned}$$

where g_t , g_ℓ and g_m are the corresponding form factors, depending, in general, on two kinematical variables, k^2 and s . In any case they can be considered as the inelastic threshold electromagnetic form factors for the S-state π^0 -production in the process $\gamma^* + d \rightarrow d + \pi^0$. This parametrization of the matrix element (which is equivalent to the corresponding description of the elastic ed -scattering, in terms of three well-known form factors) is suitable for the description of polarization phenomena for $e^- + d \rightarrow e^- + d + \pi^0$, near threshold. Consequently, a Rosenbluth fit for $e^- + d \rightarrow e^- + d + \pi^0$ (with unpolarized particles) allows us to find two quadratic combinations of form factors, namely $\sigma_L \simeq |g_\ell|^2$ and $\sigma_T \simeq |g_t|^2 + |g_m|^2$. In order to separate the g_t and g_m contributions, the measurement of the tensor analyzing power (or the final deuteron tensor polarization) is necessary, as in the case of elastic ed -scattering.

V. SPIN STRUCTURE OF THRESHOLD AMPLITUDES FOR $\gamma + D \rightarrow N + N + \pi$

In case of neutral pion production in the intermediate state, $\gamma + d \rightarrow n + p + \pi^0 \rightarrow d + \pi^0$, RE can contribute to threshold π^0 production, $\gamma + d \rightarrow d + \pi^0$ through the triplet np -intermediate state, due to the non identity of n and p , but these effects are small. This follows from the fact that the spin structure of the threshold amplitudes for the processes $\gamma + d \rightarrow p + p + \pi^-$ and $\gamma + d \rightarrow n + p + \pi^0$ are different.

In this connection we can mention that the thresholds for $\gamma + d \rightarrow p + p + \pi^-$ and $\gamma + d \rightarrow n + n + \pi^+$ processes are higher in comparison with $\gamma + d \rightarrow d + \pi^0$:

$$E_\gamma(pp\pi^-) = 145.8 \text{ MeV},$$

$$E_\gamma(nn\pi^+) = 148.5 \text{ MeV},$$

$$E_\gamma(d\pi^0) = 139.8 \text{ MeV}.$$

$\gamma + d \rightarrow p + p + \pi^-$: the spin structure of this amplitude is mainly driven by the Pauli principle. At threshold the single allowed multipole transition $E1 \rightarrow \mathcal{J}^P = 0^-$ is described by the following matrix element:

$$\mathcal{M}(pp\pi^-) = f_0 \vec{e} \cdot \vec{D}_1 \chi_2^\dagger \sigma_y \chi_1^\dagger.$$

The amplitude f_0 describes the absorption of electric dipole photons and χ_1 and χ_2 are the 2-component spinors of the final nucleons.

$\gamma + d \rightarrow p + n + \pi^0$: at threshold we have three independent multipole transitions: $E1 \rightarrow \mathcal{J}^P = 0^-$ (singlet np -production as in the case of $\gamma + d \rightarrow p + p + \pi^-$), and the following two multipole transitions: $E1$ and $M2 \rightarrow \mathcal{J}^P = 1^-$, - with triplet np -production - described by the matrix element:

$$\mathcal{M}(np\pi^0) = \chi_2^\dagger \left[\vec{\sigma} \cdot \vec{e} \times \vec{D}_1 f_e + (\vec{e} \times \vec{k} \cdot \vec{\sigma} \vec{k} \cdot \vec{D}_1 + \vec{\sigma} \cdot \vec{k} \vec{e} \times \vec{k} \cdot \vec{D}_1) f_m \right] \sigma_y \chi_1^\dagger,$$

where f_e and f_m are the corresponding multipole amplitudes. From the generalized Pauli principle, we can conclude that these amplitudes are isovector.

All polarization phenomena in $\gamma + d \rightarrow p + p + \pi^-$ can be exactly predicted, in a model independent way due to the presence of a single threshold amplitude. For example, the dependence of the cross section from the deuteron tensor polarization can be written as:

$$\frac{d\sigma}{d\omega}(\gamma \vec{d} \rightarrow pp\pi^-) = \left(\frac{d\sigma}{d\omega} \right)_0 \left[1 + \frac{1}{2} Q_{ab} k_a k_b \right],$$

where $d\omega$ is the element of the space volume for the 3-particle final state.

The presence of two threshold amplitudes, g_e and g_m , for $\gamma + d \rightarrow p + n + \pi^0$ results in different sign and value for the deuteron analyzing power:

$$\frac{d\sigma}{d\omega}(\gamma \vec{d} \rightarrow np\pi^0) = \left(\frac{d\sigma}{d\omega} \right)_0 \left[1 - \frac{1}{4} Q_{ab} k_a k_b \right].$$

At threshold, all other one-spin polarization observables for the processes $\gamma + d \rightarrow N + N + \pi$ vanish.

VI. THE PROCESS $\gamma + D \rightarrow D_S + \pi^0$

The situation with RE in the semi-coherent process $\gamma + d \rightarrow p + n + \pi^0 \rightarrow d_s + \pi^0$, with production of a 'scalar' deuteron, d_s , is very different, in comparison with the reaction $\gamma + d \rightarrow d + \pi^0$. For the process $\gamma + d \rightarrow d_s + \pi^0$, at threshold, \mathcal{J}^P takes the value 0^- and the conservation of the total angular momentum and P-parity allows the following intermediate processes (where the intermediate $pp\pi^-$ ($nn\pi^+$) have also $\mathcal{J}^P = 0^-$):

$$\gamma + d \rightarrow p + p + \pi^- (n + n + \pi^+) \rightarrow d_s + \pi^0.$$

Both rescattering contributions (Fig. 9) have the same sign: this induces a coherent increasing, instead of a cancellation (as in the case $\gamma + d \rightarrow d + \pi^0$).

Another interesting property of the considered process is connected with the isotopic structure of the corresponding amplitude. The production of d_s , with unit value of isotopic spin in the reaction $\gamma + d \rightarrow d_s + \pi^0$, is determined by the absorption of isoscalar photon (instead of isovector, for the process $\gamma + d \rightarrow d + \pi^0$). In IA, (Fig. 10), the corresponding amplitudes is proportional to the difference of the elementary amplitudes, i.e.:

$$F(\gamma d \rightarrow d_s \pi^0) \simeq (E_{0+}^{p\pi^0} - E_{0+}^{n\pi^0}).$$

As the elementary amplitudes, have different signs, one can deduce that cross section for the process $\gamma + d \rightarrow d_s + \pi^0$ is much larger in comparison with $\gamma + d \rightarrow d + \pi^0$. Therefore the ratio of these cross sections has to be very sensitive to the model chosen to determine $E_{0+}^{p\pi^0}$. For example, in the framework of ChPT, with $E_{0+}^{p\pi^0} = -1.16$ and $E_{0+}^{n\pi^0} = 2.13$ (in units $10^{-3}/m_\pi$), one can find:

$$\frac{|F(\gamma d \rightarrow d_s \pi^0)|^2}{|F(\gamma d \rightarrow d \pi^0)|^2} = \left| \frac{2.13 + 1.16}{2.13 - 1.16} \right|^2 \simeq 10.$$

In case of the dispersion relations approach (DR) [24], with $E_{0+}^{p\pi^0} = -1.22$ and $E_{0+}^{n\pi^0} = 1.19$, this ratio is much larger:

$$\frac{|F(\gamma d \rightarrow d_s \pi^0)|^2}{|F(\gamma d \rightarrow d \pi^0)|^2} = \left| \frac{1.19 + 1.22}{1.19 - 1.22} \right|^2 \simeq 6450.$$

This consideration shows that inelastic pion production, in the near threshold region, can be very large for the process $\gamma + d \rightarrow d_s + \pi^0$ and should be determined experimentally, too. A similar situation occurs for η photoproduction on the deuteron. The first experimental study of this process was done more than 25 years ago [25], but only recently it has been shown [26,27] that the cross section of the coherent process $\gamma + d \rightarrow d + \eta$ is smaller than the incoherent η -photoproduction through $\gamma + d \rightarrow p + n + \eta$.

We have shown that RE for the processes $\gamma + d \rightarrow d + \pi^0$ and $e + d \rightarrow e + d + \pi^0$, due to the intermediate processes $\gamma + d \rightarrow p + p + \pi^-$ (or $n + n + \pi^+$) $\rightarrow d + \pi^0$ is negligible in the near threshold region (when the π^0 is produced in S- and P-states). This result is obtained in a very general form, using only the symmetry properties of strong and electromagnetic interactions: the Pauli principle and conservation of P-parity and total angular momentum. It is a model independent result, and therefore it has to be verified by any model calculation. In particular, for numerical calculations, this would constitute a very important check that all contributions are properly treated and in particular double counting is avoided. The exact cancellation of RE can explain the absence of unitary cusp for the process $\gamma + d \rightarrow d + \pi^0$, at the threshold of processes $\gamma + d \rightarrow p + p + \pi^-$ and $\gamma + d \rightarrow n + n + \pi^+$. Our result about RE changes the previous interpretation of the experiment [20], concerning the evaluation of the $E_{0+}(\gamma n \rightarrow n \pi^0)$ -amplitude. The precise value of this amplitude is especially important in order to test the isotopic structure of $\gamma + N \rightarrow N + \pi$ and the predictions of ChPT [4]. The interpretation of the data about $e + d \rightarrow e + d + \pi^0$ at large momentum transfer [28], will have also to take into account this result.

We have also shown that the same general arguments predict large RE, for another possible coherent process, the photoproduction of 'scalar' deuteron, $\gamma + d \rightarrow d_s + \pi^0$, with isotopic spin $I = 1$. In this case the corresponding IA amplitude is proportional to the difference of the $\gamma + p \rightarrow p + \pi^0$ and $\gamma + n \rightarrow n + \pi^0$ amplitudes.

VIII. ACKNOWLEDGMENTS

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| | E_d | $E_{0+}^{p\pi^0}$ | $E_{0+}^{n\pi^0}$ | $E_{0+}^{n\pi^0} (IA)$ |
|------|--------------------------|------------------------|-------------------|---------------------------|
| Exp | $\pm 1.45 \pm 0.09$ [20] | -1.31 ± 0.08 [1,2] | - | 2.75 (+) -0.15 (-) |
| ChPT | -1.8 ± 0.2 [15,16] | -1.16 [4] | 2.13 [4] | |
| DR | - | -1.22 [24] | 1.19 [24] | |

TABLE I. Summary of the values of the elementary amplitudes from experiment and from model predictions, in units $10^{-3}/m_\pi$

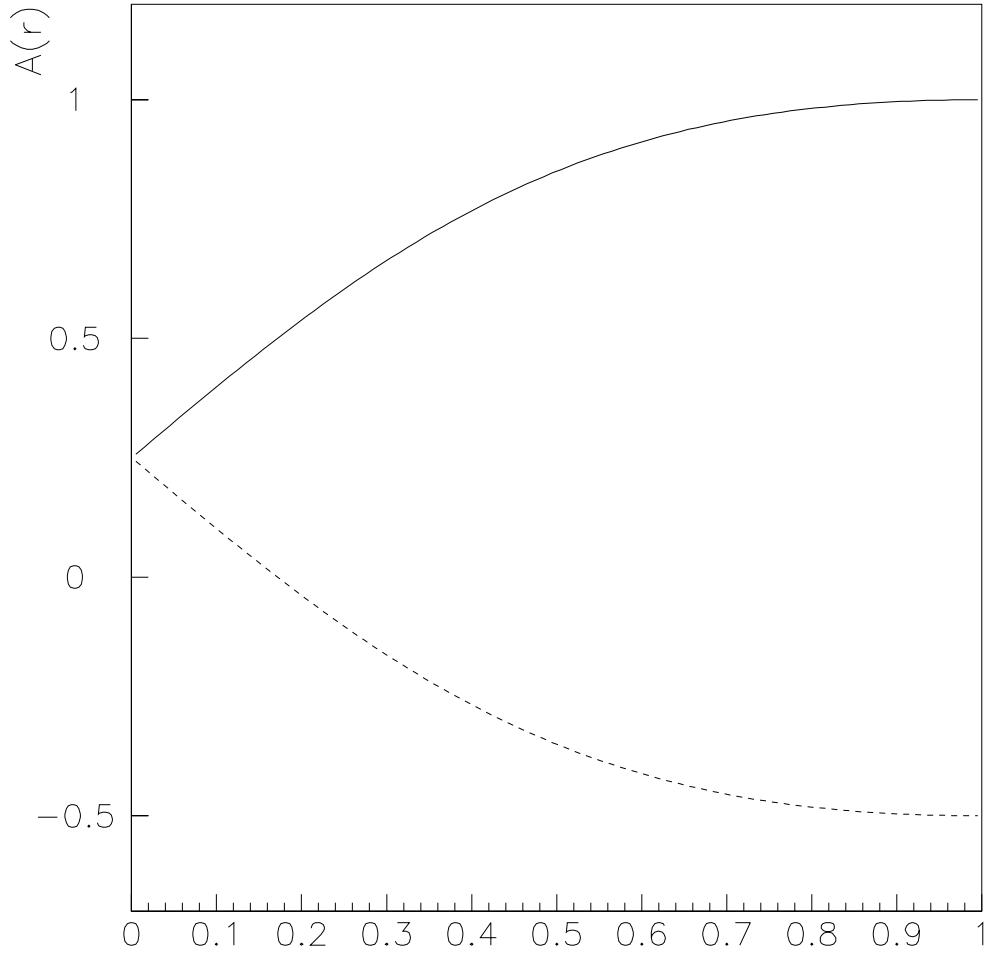


FIG. 1. Dependence of the tensor analyzing power on $r = |g_e|/|g_m|$.

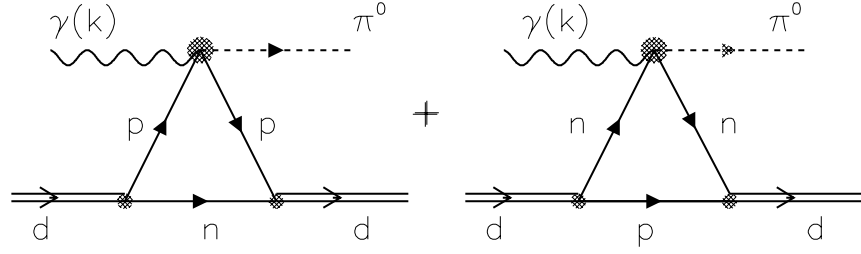


FIG. 2. Impulse approximation for $\gamma + d \rightarrow d + \pi^0$.

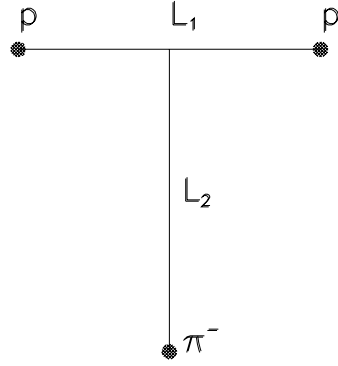


FIG. 3. Definition of the orbital momentum ℓ_1 and ℓ_2 for the $pp\pi^-$ -system.

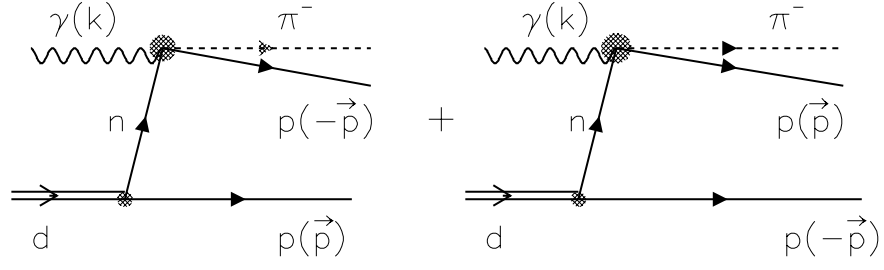


FIG. 4. Feynman diagram for $\gamma + d \rightarrow pp + \pi^-$.

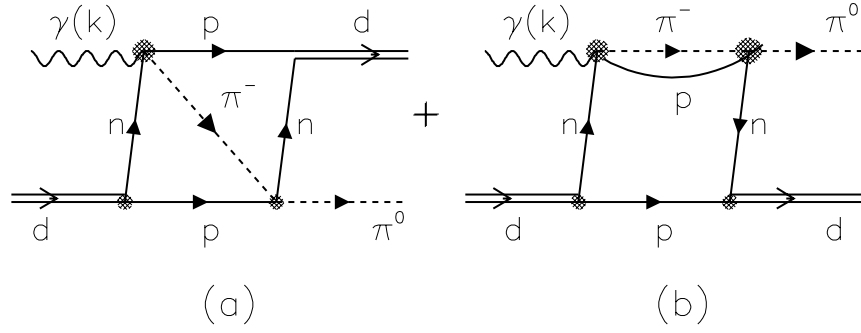


FIG. 5. Rescattering mechanism for $\gamma + d \rightarrow d + \pi^0$.

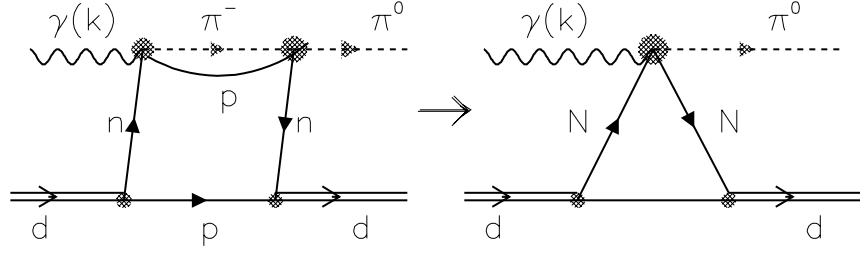


FIG. 6. Diagrams for possible double counting.

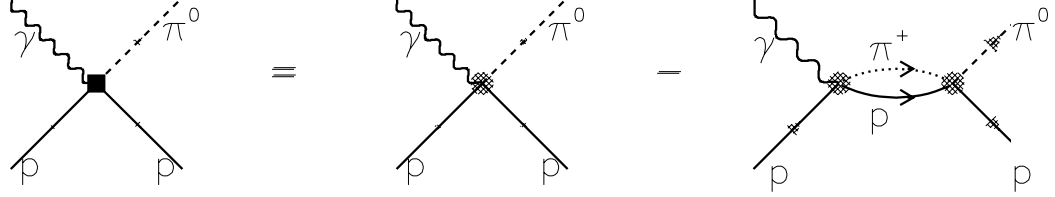


FIG. 7. Diagrammatic representation of the renormalized amplitude (solid square), whereas the solid circles represent the full amplitudes for $\gamma + p \rightarrow p + \pi^0$.

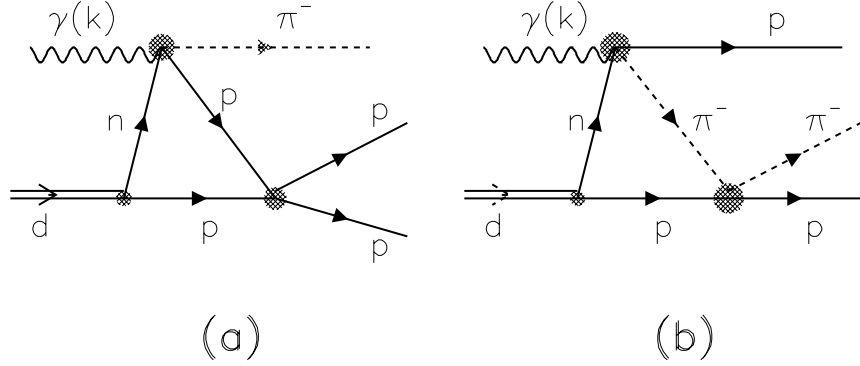


FIG. 8. Final state interaction in $\gamma + d \rightarrow \pi^- + p + p$.

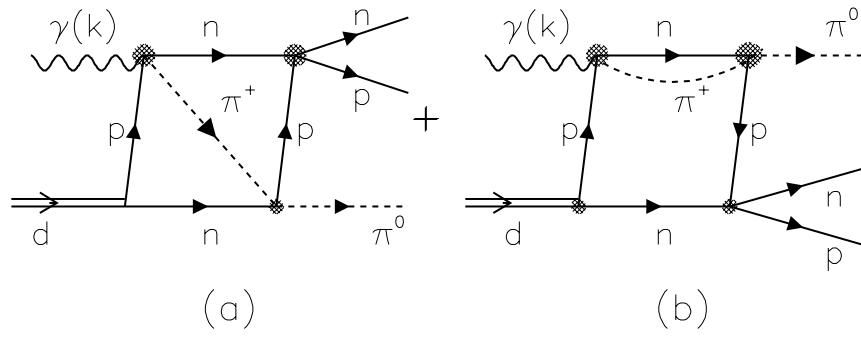


FIG. 9. Rescattering effects for the $\gamma + d \rightarrow (p + n) + \pi^0$ -process.

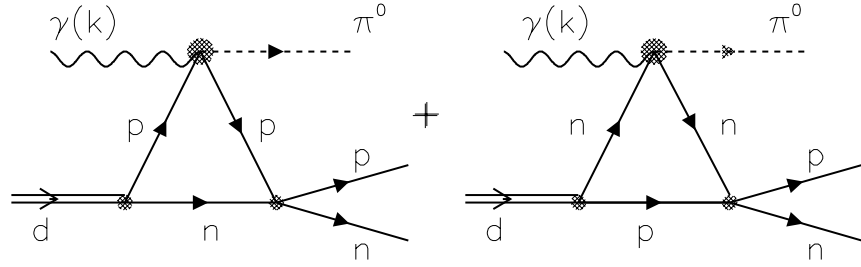


FIG. 10. Impulse approximation for the $\gamma + d \rightarrow (p + n) + \pi^0$ -reaction